Branching Ratios of the Decay Modes $\Sigma^\pm \rightarrow n+e^\pm+\nu$

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A search was made for examples of the beta-decay mode of the sigma hyperon, $\Sigma^\pm \rightarrow n+e^\pm+\nu$, among a large sample of sigmas produced by stopped K^- mesons in a propane-freon filled bubble chamber. A total of 23 000 negatively charged sigmas and 17 000 positively charged sigmas were observed which decayed via the charged π mode, $\Sigma^\pm \rightarrow n+\pi^\pm$. An electron track was distinguished from that of a π meson by the observation that the track curled up in the chamber, or curved too much, or produced a δ ray with too high an energy to have been produced by a π from a sigma decay. No Σ_β^+ decays, which are thought to be forbidden by the $\Delta S=\Delta Q$ rule, were found. A total of 11 Σ_β^- decays were found. The branching ratio for Σ_β^- decay, $\Gamma(\Sigma^- \rightarrow n+e^-+\nu)/\Gamma(\Sigma^- \rightarrow n+\pi^-)$, was found to be $(1.0_{-0.3}^{+0.4})\times 10^{-3}$. The upper limit of the ratio of the Σ_β^+ decay rate to the Σ_β^- decay rate, $\Gamma(\Sigma^+ \rightarrow n+e^++\nu)/\Gamma(\Sigma^- \rightarrow n+e^-+\nu)$, determined in this experiment was 0.4.

INTRODUCTION

THIS paper reports the results of a search for beta decays of sigma hyperons:

$$\Sigma^\pm \rightarrow n+e^\pm+\nu \quad (1)$$

among a large sample of sigma hyperons produced by stopped K^- mesons in a bubble chamber filled with a propane-freon mixture. There were two purposes for this experiment: the first, to find out the extent to which the $\Delta S=\Delta Q$ rule[1] is violated in sigma-beta decays by comparing the number of Σ_β^- (allowed) to the number of Σ_β^+ decays (forbidden by the rule and still unobserved); the second, to measure the branching ratio $\Gamma(\Sigma^- \rightarrow n+e^-+\bar{\nu})/\Gamma(\Sigma^- \rightarrow n+\pi^-)$ as a test of the theory of Feynman and Gell-Mann.[1]

According to the $\Delta S=\Delta Q$ rule, decays such as the following are forbidden:

$$\begin{aligned} &K^0 \rightarrow e^-+\pi^++\bar{\nu} \\ &\bar{K}^0 \rightarrow e^++\pi^-+\nu \\ &\Sigma^+ \rightarrow e^++n+\nu \\ &\Sigma^+ \rightarrow \mu^++n+\nu \\ &K^+ \rightarrow e^-+\pi^++\pi^++\bar{\nu}. \end{aligned} \quad (2)$$

The first evidence for the violation of this rule was found in a study of the time dependence of the K_{e3} decay mode of neutral K mesons.[2,3] This result immediately suggested looking for direct violations of the rule among decays of other strange particles.

There is no reason to expect the extent of the violation to be the same for all particles because of the different currents involved. If only vector or axial-vector couplings between the strongly interacting particles in the decay are assumed, the K_{e3}^0 decays are pure vector interactions, whereas the $\Sigma_\beta^\pm$ decays and the decay mode $K^+ \rightarrow e^++\pi^++\pi^-+\nu$ involve both vector and axial-vector currents. The decay mode $K^+ \rightarrow e^-+\pi^++\pi^++\bar{\nu}$ involves primarily an axial-vector current.

Other experiments[4–6] have already shown that the branching ratios for leptonic decays of sigmas and lambdas predicted by Feynman and Gell-Mann are too low by an order of magnitude. Two very recent experiments[5,7] indicate that the $\Delta S=\Delta Q$ rule is much less violated in the leptonic sigma decays and K_{e4} decays than in the leptonic K^0 decays. The only evidence for a direct violation of the rule is a single example of the decay mode $\Sigma^+ \rightarrow \mu^++n+\nu$ reported by Barbaro-Galtieri *et al.*[8]

In the experiment reported here, no Σ_β^+ decays and $11\Sigma_\beta^-$ decays were found. This data supplements that of the Cern-Maryland experiment[5] on the validity of the $\Delta S=\Delta Q$ rule for Σ_β decays. Furthermore, a careful study of electron detection efficiencies, absolute abundances of Σ hyperons, and contaminations from background events in this experiment has allowed a determination of the Σ_β^--decay branching ratio with an error of about 40%.

The use of a heavy liquid, rather than liquid hydrogen, permitted the detection of electrons throughout the energy spectrum of electrons from Σ_β decays with an efficiency of approximately 50%. Therefore, the results are very insensitive to the shape of the electron energy spectrum. However, the elimination of

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[5] R. A. Burnstein, T. B. Day, A. J. Herz, B. Kehoe, B. Sechi-Zorn, N. Seeman, G. A. Snow, H. Courant, H. Filthuth, P. Franzini, G. G. Glasser, A. Minguzzi-Ranzi, A. Segar, and W. Willis, Proceedings of Sienna Conference, 1963 (to be published).
[6] R. P. Ely, G. Gidal, G. E. Kalmus, L. O. Oswald, W. M. Powell, W. J. Singleton, F. W. Bullock, C. Henderson, D. J. Miller, and F. R. Stannard, Phys. Rev. Letters **131**, 868 (1963).
[7] R. W. Birge, R. P. Ely, G. Gidal, G. E. Kalmus, A. Kernan, W. M. Powell, U. Camerini, W. F. Fry, J. Gaidos, R. H. March, and S. Natali, Phys. Rev. Letters **11**, 35 (9163).
[8] A. Barbaro-Galtieri, W. H. Barkas, H. H. Heckman, J. W. Patrick, and R. M. Smith, Phys. Rev. Letters **9**, 26 (1962).

background events and the identification of sigmas were much more difficult problems in this experiment than in a hydrogen bubble chamber experiment.

EXPERIMENTAL PROCEDURES

The bubble chamber photographs used for this experiment were the same ones used in the recent determination of the Λ_β-decay branching ratio.[6] Photographs were taken of about 500 000 K^- mesons which stopped in the Berkeley 30-in. heavy liquid chamber. The chamber was filled with a propane-freon mixture (45% CF_3Br, 55% C_3H_8, by volume) which had a radiation length of 23 cm and a density of 0.89 g/cm^3. Further details about the beam and chamber may be found in the report on the Λ_β-decay experiment.

Production of Sigmas

The K mesons produced about 250 000 observable lambdas and 40 000 sigmas which decayed into charged π mesons. Nearly all of the sigmas were produced by the four reactions shown in Table II. The production of a charged Σ and a π^0 was very rare: materialized gamma rays coming from the production origin were observed in only 3% of all the Σ productions.

Identification of sigma productions rested mainly on the visual observation of the short track of a heavily ionizing particle emerging from a K interaction origin which decayed into a lightly ionizing particle (usually a π meson). Since the K mesons interacted mostly with heavy nuclei, one could not identify sigmas nor determine their momenta from production kinematics. A more precise description of acceptance criteria for sigmas will be found below.

It was usually impossible to obtain the Σ momentum from decay kinematics because of the large error in the momentum of the π measured from curvature. The momentum distribution of these sigmas could, in principle, have been determined but was tremendously complicated by Σ^- captures at rest and Σ^+ decays at rest. Fortunately, it was not necessary to know the distribution well in this experiment. An average momentum at decay of 300 MeV/c for the Σ^- hyperons and 250 MeV/c for the Σ^+ hyperons was consistent with the length distributions and decay angular distributions. When an approximate production momentum distribution was needed, the Λ momentum distribution[6] was used, appropriately compressed to take into account the smaller energy release in the production of sigmas.

The sigmas tended to be quite short: about 80% of them were less than 1 cm long. The length distributions for three of the production categories are shown in Fig. 1.

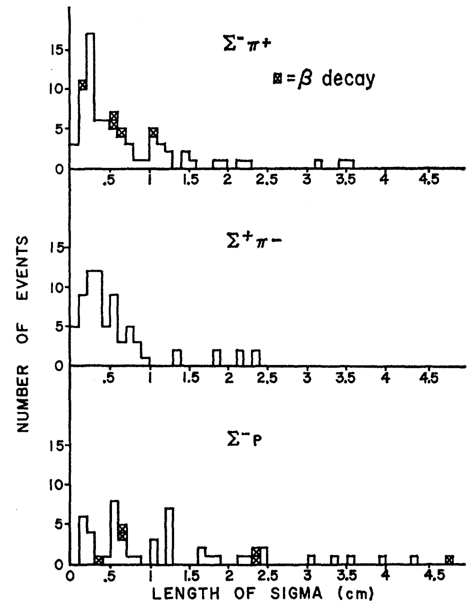

FIG. 1. Experimental length distribution of charged sigmas observed decaying into charged mesons, for three production final states. The marked boxes are the 11Σ decays.

Scanning

Scanners were instructed to record those sigma decays in which the track of the charged decay product showed possible evidence of being that of an electron, independently of the fact that it came from a sigma. Specifically, a track was recorded as possibly that of an electron if any one of the following characteristics was observed: (1) The track curled up and stopped in the chamber, or the length of the line joining the origin of the track to successive points on the track passed through a maximum before the last visible bubble of the track.[6] (2) The sagitta of the track or of the last half of the track exceeded the sagitta of an ideal (no multiple scattering) stopping π meson by more than one "standard deviation." The precise statistical meaning of this "standard deviation" will be discussed later. (3) A δ ray longer than 5 mm (corresponding to a kinetic energy of 1.1 MeV) was attached to the track.

Events in these categories will be referred to as curl up, curvature, and δ-ray candidates, respectively. Events in which the possible electron became highly ionizing and stopped in the chamber, or scattered from a nucleus leaving a visible proton recoil, were rejected as π mesons by the scanners.

About four candidates per roll were recorded, most of which were curvature candidates between 1 and 2 standard deviations or δ-ray candidates with 1 to 2.5-MeV δ rays, none of which became a real candidate for a beta decay. Quite intentionally, there were about equal numbers of curvature and δ-ray candidates, in order to keep the efficiency high in both categories.

With these scanning criteria, any examples in the film of the decay $\Sigma^{\pm}\rightarrow\Lambda+e^{\pm}+\nu$ should have been found. However, there is a possibility that there was a bias in the scanning and editing against such decays, especially if the sigma was positive. The lambdas from these decays would have tended to be quite slow and often would have decayed very close to the end of the sigma. The apparent three-prong vertex probably would not have been noticed as a Σ decay by the scanners. Examples of the positively charged mode of this decay would have been rejected as a $\pi^+ \to \mu^+ \to e^+$ chain in which the Λ came from the production origin, if the electron had a momentum less than 53 MeV/c (see E4, below) and the Λ pointed towards the origin. These two conditions usually would have been fulfilled because of the low Q value of the decay.

ACCEPTANCE CRITERIA AND BACKGROUNDS

Each event recorded by the scanners was examined and measured on the scanning table by physicists to determine whether the event was acceptable as a β decay. Candidates which were not obviously rejectable were studied more carefully on photographic enlargements and measured with the digitized microscope.

Whenever possible, backgrounds were eliminated or reduced to a tolerable level by drawing suitable acceptance cutoffs. The acceptance criteria may be divided into two nearly independent groups: requirements for the production vertex and track of the sigma and requirements for the electron.

Σ Acceptance Criteria

The following acceptance criteria for sigma production were also adhered to in the determination of the absolute abundances (Table II) of mesonic sigma decays.

(S1) The production origin was required to be inconsistent with a K scatter. This requirement was necessary to eliminate the background of $K-p$ scatters followed by a K_{e3} decay or by the production of an asymmetric Dalitz pair, in which the track of the scattered K looked like that of a Σ. The observation of a production π eliminated this possibility, since the beam momentum was below the threshold for the process $K+p \to K+\pi+p$. The process $K+\text{nucleus} \to K+p+p+$(unseen fragments) was found to be so rare that all apparent sigmas with two or more visible protons at the production origin were accepted as sigmas. In the $\Sigma^\pm p$ productions with only one visible (kinetic energy >7 MeV) proton, this background was eliminated by observing that the outgoing energy was too much greater than the energy of the incident K meson to allow the hypothesis of $K+p \to K+p$:

$$T_{K\,\text{in}} < T_{K\,\text{out}} + T_p. \tag{3}$$

The kinetic energies of the proton (T_p) and outgoing sigma, hypothesized to be a K for this test ($T_{K\,\text{out}}$), were measured by range. Unfortunately, the kinetic energy of the incident K ($T_{K\,\text{in}}$) could not be measured even to 50% accuracy by any method. Therefore, the incident K meson was assigned an energy equal to a practical upper limit which was determined in one of two ways: (a) If the ionization of the K was consistent with that of a stopped K, the upper limit was taken to be 60 MeV. (b) If the K was clearly in flight, the upper limit was obtained by assuming that the incident K had a residual range which would have carried it to a line 25 cm below the center of the chamber. Only 2% of the K mesons traveled beyond this line. The effect of these selection criteria was that about 55% of the real $\Sigma^\pm p$ events were rejected as possible K scatters.

The remaining background from K scatters in this selection of events was determined from an experimental study of the abundance of K scatters. The abundance of K scatters which would have been acceptable as sigmas, had the K decayed, was found to be 0.75×10^{-3} per incident K for single-proton scatters and 1.6×10^{-3} per incident K for two-proton scatters. The average probability for a K_{e3} decay was calculated to be 0.22×10^{-3} per K scatter. Detection and scanning efficiency factors were included in this calculation. Since 470 000 K^- mesons were observed in the experiment, the probability of observing one such simulated Σ_β decay in the whole experiment was 0.24 for negatively charged decays. For positively charged decays, in which the background comes only from asymmetric Dalitz pairs, the probability was only 0.02.

(S2) The length of the track of the sigma, as measured in space from a microscope measurement, was required to be greater than 0.1 cm.

(S3) The angle between the tracks of the sigma and the electron was required to be large enough to rule out the possibility that the electron came directly from the origin (in which case, the 'sigma' was probably a proton accidentally colinear with an electron). This angle depended on the length of the sigma and was typically about 10°.

(S4) The ionization of the sigma was required to be great enough to make the possibility very unlikely that the track of the sigma was actually that of an electron which scattered, or that of a π meson which underwent charge exchange, producing two gamma rays, one of which produced a Compton electron within 2 mm of the charge exchange point. In effect, this requirement allowed no gaps in very short sigmas (0.1 to 0.3 cm) and only about 4 per cm in longer sigmas.

The amount of background expected from electron scatters was estimated from the number of events observed in which an electron came from an origin which was otherwise acceptable as a sigma origin (about 300 events). The probability that one of these electrons would be Coulomb scattered through an angle greater than 15° and have no gaps within 0.2 cm of its origin was calculated to be 0.05. Similarly, the probability of one charge-exchange sequence, described above, occurring in the experiment was found to be approximately 0.03, by using typical charge-exchange cross sections[9] and Compton electron cross sections.[10]

[9] H. Bethe and F. de Hoffmann, *Mesons and Fields* (Row Peterson & Company, New York, 1955), Vol. II, p. 27.

[10] B. Rossi, *High Energy Particles* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1952), p. 178.

(S5) The sigma was required to be 'moderately' straight. This was a subjective test applied to the longer (>0.6 cm) sigmas and consisted of the requirement that the sigma appear more like a stopping proton than a stopping π or K. Its only effect was to eliminate a few negative, and quite a few positive, $\pi-\mu-e$ chains in which the $\pi-\mu$ track was mistaken for a sigma by the scanner.

Unfortunately, the subjective nature of this test made it difficult to estimate the remaining background. The abundance of slow π^+ mesons, which always stop and decay, was large enough that all apparent Σ_β^+ decays with electron momenta less than 53 MeV/c, which is the maximum for electrons from μ decay at rest, had to be eliminated (criterion E3, below). To determine the background from negative π mesons, which decay only in flight, a careful study was made of the abundance of π^- mesons produced at acceptable origins and of their momentum spectrum. A computer program was written to compute the decay probability for this sample of π mesons as a function of the total length of the $\pi-\mu$ track. The probability that the $\pi-\mu$ track would appear "moderately" straight was evaluated in a semi-empirical fashion. The resultant probability for the occurrence of a spurious Σ_β^- decay in the whole experiment was 0.04 for an event with a production π^+ and 0.19 for an event with only protons from the origin. A detection and scanning efficiency of 0.5 was used in the calculation.

(S6) The dip of the sigma was required to be less than 60°.

(S7) Events in which the end of the track of the sigma was closer than 0.3 cm to the top or bottom glass were eliminated in order to assure that no other charged particles emerged from the end of the sigma.

(S8) Those $\Sigma^{\pm}\pi^{\mp}$ productions in which the sigma and the incident K were sufficiently colinear so that it could not be determined which vertex was the production vertex were eliminated.

(S9) The gap between the end of the track of the sigma and the first bubble of the track of the electron was required to be less than 0.2 cm.

(S10) The extrapolated trajectory of the track of the electron was required to cross the track of the sigma no more than 1 bubble width from the end of the track of the sigma. A bubble width was taken to be the width of the track of the sigma and was usually 0.05 cm.

This last criterion eliminated most of the possible background from the decay $\Sigma^+\rightarrow p+\pi^0$ in which the π^0 produced an asymmetric Dalitz pair and in which the proton was very short. From the assumption that $\frac{1}{10}$ of all $\Sigma^+\rightarrow p+\pi^0$ decays had protons shorter than 0.05 cm, or which went backwards from the decay vertex, it was calculated that the probability of one such spurious event was 0.07.

Criteria (S9) and (S10) together reduced the background from the process Σ^-+nucleus $\rightarrow\Lambda$ (Σ^- capture at rest), followed by $\Lambda\rightarrow n+\pi^0$ very near the end of the sigma, in which a gamma from the π^0 produced a Compton electron which appeared to come directly from the end of the sigma.

By averaging the stopping probability over an approximate momentum distribution for the sigmas at production, the number of negative sigmas longer than 0.1 cm which stopped and were captured was estimated to be 1.0 times the number of mesonic decays observed. The fraction of the lambdas which decayed into neutral π mesons within 0.1 cm of the Σ^--capture origin was calculated to be 0.026, from an estimate for the momentum spectrum of these lambdas.[11] The probability of one of the two 85-MeV (on the average) gamma rays producing a Compton electron[10] within 0.2 cm was calculated to be 1.0×10^{-3}. By multiplying these factors by the total number of mesonic Σ^- decays in the experiment and a detection and scanning efficiency of 0.40, the probability of observing one such background event was found to be 0.24.

The only one of these selection criteria which eliminated a large number of events which were really sigmas was (S1). The other criteria had the effect of rejecting about 12% of the mesonic sigma decays found in the scanning.

Electron Acceptance Criteria

The acceptance criteria for electrons were of two varieties: those which applied to all candidates and those which defined the acceptance cutoffs in each of the three categories (curl up, curvature, and δ ray). The first 4 applied to all candidates.

(E1) The dip of the electron was required to be less than 60°, since ionization measurements became unreliable for tracks with greater dip angles.

(E2) The ionization of the electron was required to be within the statistical error of 1.4 times minimum, or less, everywhere along the track. Relative ionization was measured by bubble counting in 3-cm segments. This requirement eliminated all tracks containing a $\pi\rightarrow\mu\rightarrow e$ decay sequence. Since virtually all μ decays occur at rest, the change in ionization at the decay point was obvious. Slow pions which decayed into muons which were at rest in the laboratory were also eliminated by this requirement.

(E3) In that portion of the track for which the momentum by curvature was greater than 100 MeV/c, no kinks greater than 30° were allowed. This requirement eliminated most of the background from normal mesonic sigma decays in which the π^- underwent charge exchange, producing two gammas, one of which produced a Compton electron within a few millimeters of the point of charge exchange. The probability that a real Σ_β decay would have been rejected because of this criterion was calculated and found to be negligible.

[11] The spectrum was assumed to be the same shape as that for lambdas produced by K mesons (Ref. 6), but peaked at 290 MeV/c, which is the momentum of a lambda produced by Σ^- capture by a free proton.

The probability of a charge exchange simulating an acceptable Σ_β decay in the whole experiment was calculated to be 0.03. This result is based on differential cross sections for charge exchange at two π momenta (150 and 220 MeV/c),[9] the probability for producing a Compton electron within 0.5 cm of the charge exchange point[10] (evaluated at the average gamma energy), an average track length of 15 cm, 23 000 observed mesonic Σ^- decays, and a detection and scanning efficiency of 0.30. The expected background from charge exchange is at least ten times smaller in the case of mesonic Σ^+ decays, since the background comes only from asymmetric Dalitz pairs or conversion pairs.

(E4) In the candidates for Σ_β^+ decays, the electron momentum was required to be greater than 53 MeV/c in order to eliminate $\pi-\mu-e$ chains (see S5). Specifically, this meant either that the electron range was required to be greater than 32 cm[12] or that the momentum by curvature was required to be sufficiently greater than 53 MeV/c. About 25 candidates (all of which satisfied criterion S5) were rejected with momentum distinctly below 53 MeV/c. From this number and from the momentum spectrum of electrons from μ decay,[13] it was determined that 1.5 standard deviations above 53 MeV/c, which was about 95 MeV/c in a typical case, was sufficiently greater than 53 MeV/c to reduce the $\pi-\mu-e$ background probability to 0.2. The application of this cutoff eliminated the remaining 5 candidates for Σ_β^+ decay. This cutoff was the major source of a much lower detection efficiency for Σ_β^+ decays than for Σ_β^- decays.

Curvature Candidates

There were no further cutoffs for the curl-up candidates. The acceptance cutoffs for the curvature candidates were as follows.

(E5) The electron track was required to have a length greater than 10 cm.

(E6) The sagitta of the track was required to exceed the magnetic sagitta expected for a π, assumed to have a momentum of 85 MeV/c at the last visible bubble, by four times the root-mean-square projected sagitta (hereafter referred to as the standard deviation, or S.D.) expected from multiple scattering for the same π. The standard deviation, as calculated from multiple scattering theory,[14] is given by the equation:

$$\langle S^2\rangle_{\text{proj}}^{1/2}=\frac{2.16L^{3/2}}{P\beta L_{\text{rad}}^{1/2}}=\frac{0.45L^{3/2}}{P\beta}, \tag{4}$$

in which L is the true track length in cm, L_{rad} is the radiation length, and $P\beta$ is the momentum times velocity/c of the track in MeV/c. A study of 200 long, stopping μ mesons showed that the distribution of sagittas was Gaussian (at least out to 2 S.D.) and that the standard deviation was given by Eq. (4) with $P\beta$ evaluated at the center of the track, with 6% uncertainty.

The effect of single-scattering, which normally introduces a large non-Gaussian "tail" to the multiple scattering distribution, can be shown to be negligible beyond 3 S.D. because of the fact that large kinks (e.g., greater than 10° in a track with $P\beta=90$ MeV/c) could always be seen. Therefore, the probability of a π exceeding 4 S.D. is the remaining fraction of the area under the Gaussian curve beyond 4 S.D., or 1/32 000. Actually, the probability that a π from a Σ decay would exceed 4 S.D. is about twice as small, since these π mesons left the chamber with an average momentum of 140 MeV/c, not 85 MeV/c. Below 85 MeV/c, a π was distinguishable from an electron by ionization. The probability of a π from a Σ^- or Σ^+ decay exceeding 4 S.D. once in the experiment is, therefore, about 0.14 or 0.09, respectively.

The sagittas of all candidates found by the scanners were measured on the scanning table and compared with the standard deviation marks on one of a series of π templates according to the dip of the track. All candidates above 2 S.D. were measured on the microscope and evaluated in the spacial reconstruction program, which removed optical distortions and magnetic field variations. About 20 candidates lay between 2 and 3 S.D., 3 between 3 and 4 S.D., and 2 above 4 S.D. The latter two events were accepted as Σ_β^- decays.

δ-Ray Candidates

(E7) The δ ray was required to have a kinetic energy greater than 2.5 MeV.

(E8) The δ ray was required to be well attached to the track, which meant that (a) the angle between the δ ray and the track was less than 50°; (b) the first bubble of the δ ray distinguishable from the main track was within 0.1 cm of the track, measured along the trajectory of the δ ray; (c) the trajectory of the δ ray intersected the main track in space, within the measurement errors.

These two criteria were necessary to reduce the large background which came from the many low-energy Compton electrons in the chamber. If the first bubble of one of these Compton electrons occurred close enough to the track, it would simulate a δ ray. From the three requirements (E8), it was calculated that the effective volume of the average track (15 cm in length) within which a Compton electron would have to begin in order to simulate a δ ray was 0.02 cm^3. The abundance of stray Compton electrons in the chamber decreased rapidly as the energy increased. Above 2.5 MeV, the average density was found to be 0.87×10^{-3} electrons per cm^3. Therefore, the probability of observing such a

[12] An average value of 1.65 MeV/cm was used for dE/dx in this momentum region. I am indebted to David Miller, University College, London, for the calculation of the energy loss for electrons in this liquid.
[13] B. Rossi, Ref. 10, p. 90.
[14] W. T. Scott, Phys. Rev. **76**, 212 (1949).

TABLE I. Summary of backgrounds.

Background	Reference to discussion[a]	Probability of one event in experiment $\Sigma^-\pi^+$	Σ^-p	$\Sigma^+\pi^-$	Σ^+p	Restrictions
K scatter $\rightarrow K_{e3}$	S1		0.24		0.02	
electron scatterers	S4	0.03	0.05	0.01		
Σ^- capture	S10	0.14	0.10			
charge exchange	E3	0.02	0.01			
$\Sigma^+\rightarrow p+\pi^0$	S10			0.07		
$\pi^-\rightarrow\mu^-\rightarrow e^-$	S5	0.04	0.19			$P_{e^-}<55$ MeV/c
π curves >4 S.D.	E6	0.08	0.06	0.09	0.01	curvature events
δ really Compton e^-	E8	0.22	0.16	0.25	0.03	δ-ray events
δ from π or μ	E9	0.10	0.07	0.11	0.02	δ-ray events
Totals		0.63	0.88	0.53	0.07	

[a] See lettered paragraphs in text.

background event is 0.66, since the total track length of π mesons from $\Sigma^{\pm}$ decays was 600 000 cm and the scanning efficiency for δ rays was 65%.

The observation of a δ ray with kinetic energy greater than 2.5 MeV is not sufficient to prove that the track was that of an electron. A δ ray with kinetic energy T_δ can be produced by any π or μ with a momentum greater than a minimum given by the equation

$$T_\delta=\frac{2m_eP_{\min}{}^2}{M^2+2m_eW_{\min}+m_e{}^2}, \qquad (5)$$

in which $P_{\min}$ and $W_{\min}$ are the minimum momentum and total energy of the π or μ necessary to produce the δ, M is the mass of the π or μ, and m_e is the mass of the electron. This equation follows from energy and momentum conservation in the collision of a π with a stationary electron. The π mesons from sigma decay in this experiment decayed in flight into muons frequently enough that the hypothesis that the track was a μ had to be considered. Elimination of the possibility that the δ ray was produced by a μ from a π decay nearly automatically eliminated the possibility that it was produced by a π, since a μ can produce a δ ray with twice as much energy as a π with the same momentum. These considerations led to the last requirement:

(E9) The momentum (at the δ ray) of the track, assumed to be a μ, was required to be less than the minimum μ momentum, $P_{\mu\,\min}$ (Eq. 5), necessary to produce the δ ray. The momentum of the track was determined in one of two ways. First an upper limit to the momentum from the assumption that the event was a $\Sigma\rightarrow\pi$, $\pi\rightarrow\mu$ chain was established from the kinematics of the two decay processes. An upper limit to the Σ momentum at the decay vertex was obtained from energy conservation at the production vertex.[15] The second method of momentum measurement was by curvature. However, this momentum was disregarded if it was within 2 standard deviations (Eq. 4) of the minimum momentum necessary for a π to produce the δ ray ($P_{\pi\,\min}$). Otherwise, the event was accepted as a β decay if either of the two momenta was lower than $P_{\mu\,\min}$.

Because of the 2.5-MeV minimum δ-ray energy (E7), the background from pions and muons comes only from pions with momenta greater than 225 MeV/c and muons with momenta greater than 158 MeV/c. From the sample of measured mesonic Σ decays, it was estimated that about 15% of the π mesons had momenta greater than 225 MeV/c. The average probability that a π would produce a δ ray with kinetic energy greater than 2.5 MeV and have a momentum by curvature equal to, or less than, $P_{\mu\,\min}$ and at least 2 standard deviations below $P_{\pi\,\min}$ was calculated to be 0.9×10^{-5} per Σ_π decay. An analogous calculation indicated that the probability was approximately the same that a π would decay in flight into a μ and that the μ would satisfy the same conditions. From these numbers, the probability for the observation of one such accidental event in the experiment was found to be 0.30. The background probability would be much smaller for events established by the kinematic upper limit to the μ momentum, because this upper limit is always an overestimate of the momentum.

The energy of the δ ray was measured from the range in space using Feather's rule in aluminum,[16] scaled slightly for the different properties of this liquid. Care was taken not to overestimate the dip of the δ ray. The error in the measured range was estimated to be about 0.10 cm for δ rays between 1 and 3 cm long. The tests for attachment (E8) were carried out both by the use of photographic enlargements and by examination of the space-reconstructed points from the optical reconstruction program.

Two β decays were identified by δ rays. In both cases, it was the curvature measurement of momentum, rather

[15] See paragraph (S1). If the event showed no evidence of a production π, the Q value of the reaction was assumed to be that of two-nucleon production, $K^-+p+p\rightarrow\Sigma^-+p$.

[16] B. H. Willis and C. V. Stableford, University of California, Lawrence Radiation Laboratory Report UCRL-2426 Rev., 1956, Vol. 2 (unpublished).

than the kinematic upper limit, which established the event.

Summary

The backgrounds mentioned above are summarized in Table I. The probability of observing a background event in each of the most common production classes is tabulated. The grand total of these backgrounds is $1.5\Sigma^-$ events and $0.6\Sigma^+$ events. The largest background occurs in the δ-ray events.

One other process which is considered background in this experiment is the decay mode $\Sigma^- \rightarrow \Lambda + e^- + \bar{\nu}$ in which the Λ decayed into neutral particles. Since no Σ decay of this type was observed in which the Λ decayed into charged particles, it may be said that the probability that one of the observed events is such a decay is less than 0.5. However, there was a possible scanning bias against such decays if the Λ did decay via the charged mode (see above). If the branching ratio for this decay mode measured by the Cern-Maryland collaboration[5] is used, the probability is 0.6 that one of the events found in this experiment is a $\Sigma^- \rightarrow \Lambda + e^- + \bar{\nu}$ decay. This probability raises the amount of background contamination to 2 Σ^- events.

Many of these background calculations have an uncertainty of about a factor of two. Furthermore, all the background probabilities are quite sensitive to the choice of cutoffs made. In some cases, it was rather difficult to apply the acceptance criteria rigorously to the candidates. Two candidates were on the borderlines defined by these criteria. These two uncertainties lead to an uncertainty of about a factor of two in the total number of background events expected.

Accepted Events

A total of $11\Sigma_\beta^-$ decays remained which satisfied the acceptance criteria. There were seven curl-up candidates, two δ-ray candidates, and two curvature candidates. No Σ_β^+ decays were found. The laboratory energies of the electrons, measured by the method of Behr and Mittner,[17] are displayed in Fig. 2. Subtraction of the expected background contamination reduces the total number of events to nine examples of the decay mode $\Sigma^- \rightarrow e^- + n + \bar{\nu}$.

DETERMINATION OF THE BRANCHING RATIO

The accuracy of the branching ratio is limited mostly by the small number of events, which introduces an error of about 25%. Therefore, the other factors which enter into the branching ratio have been determined only to 10% accuracy. These other factors are the number of mesonic Σ decays in the film, the detection efficiency of the chamber for electrons, and the scanning efficiency.

TABLE II. Numbers of acceptable mesonic Σ decays in the experiment.

Production reaction	Number of events	Totals
$K^- + \text{nucleus} \rightarrow \Sigma^- + \pi^+$	13 000±1300	
$\rightarrow \Sigma^- + \text{protons}$	9800±1250	
Total Σ^-		22 800±1900
$\rightarrow \Sigma^+ + \pi^-$	14 700±1300	
$\rightarrow \Sigma^+ + \text{protons}$	2500±600	
Total Σ^+		17 200±1500

The Number of $\Sigma\pi$ Decays

Seven rolls of film were scanned about six times by different scanners for all $\Sigma^\pm \rightarrow \pi^\pm + n$ decays. All events were measured on the digitized microscope. The sigma acceptance criteria (S1 through S10) were applied and the number of acceptable Σ decays determined. The number of Λ decays on each roll, obtained from the Berkeley and London scanning records, was then used as a measure of the beam intensity in these seven rolls and in the rest of the film and the number of Σ decays in the whole experiment determined by scaling. The results are summarized in Table II.

The errors in Table II are the result mainly of the statistical error in the number of sigma decays counted. Also included in the error is a 5% uncertainty arising from the subjective judgment sometimes involved in applying the acceptance criteria.

Since the acceptance criteria for the production vertex and the track of the Σ were the same for both mesonic and β decays, scanning biases and detection efficiencies relating to the properties of the track of the sigma should be the same for both the mesonic and the β decays. Therefore, in calculating the rate for Σ_β^- decay, one needs to correct the ratio of the number of β decays to the number of mesonic decays only for the detection efficiency of the chamber for electrons compared with that for π mesons, and for the scanning efficiency for β decays relative to that for mesonic decays.

The scanning efficiency in the determination of the number of mesonic Σ decays was taken to be 100%, on the assumption that an event with some peculiar topology not found in six scannings of a roll would never be found, even if the π were replaced by an electron.

Similarly, the detection efficiency of the chamber for mesons from sigma decays may be taken to within a few percent of 100%, since the π was not required to stop in the chamber. Several corrections which were considered, such as the loss of events in which the track of the π was very steep or in which $\theta_{\Sigma\pi}$ was very small (see S3), either were the same for the mesonic and the

[17] L. Behr and P. Mittner, in *Proceedings of the 1962 Conference on Instrumentation for High-Energy Physics at Cern*, edited by F. J. M. Farley and M. E. Meyer (North-Holland Publishing Company, Amsterdam, 1963), p. 446.

β decays or changed the efficiency only 1%. Background contaminations were similarly small.

Scanning Efficiency

The efficiency with which the film was scanned for Σ_β decays was determined on the basis of about 75 events which were either acceptable Σ_β decays or close imitations thereof. Candidates with δ rays of energy less than 2.5 MeV and curvature candidates below two standard deviations, which constituted 95% of the events recorded by the scanners, were not used. A rescanning of those rolls containing these 75 candidates, in which the scanners were not aware that they were rescanning, was undertaken during the last few months along with the first scanning of other rolls. These rolls were scanned by three or four different scanners. From this, and earlier, rescanning, an efficiency was determined for each scanner in each category. The assumption was made that every electron satisfying criteria (E1) through (E9) had an equal probability of being found. The efficiency with which the whole experiment was scanned was calculated to be $(79\pm5)\%$ for the curl-up and curvature events, $(66\pm9)\%$ for the δ-ray events. The calculation included the appropriate correction for the fact that 20% of the film was scanned twice.

These numbers would be wrong if electron tracks with certain topologies were consistently missed, if these topologies had no analogy in the π tracks of the mesonic sigma decay sample. There was some suspicion that very high-energy δ rays (>6 MeV) may have been missed. If this scanning bias actually existed, the true scanning efficiency for δ rays would be twice as low as the number quoted above.

Detection Efficiency

A Monte Carlo program similar to the one used in the Λ_β-decay experiment[6] was written to compute the efficiency of the chamber for detecting electrons by the three different methods (curl up, curvature, and δ ray).

The program received as input the initial angles and origin coordinates of each of 180 π mesons from a random sample of mesonic sigma decays. In outline, the program replaced the π by an electron and then generated its trajectory 20 times, making different random choices for the initial energy and radiation loss per centimeter each time. Each track was then analyzed in exactly the same way as the real candidates (E1 and E4–E6) to see if it was an acceptable curl-up or curvature event. If it was not acceptable, then the probability that it would produce a δ ray larger than the maximum allowed for a μ (criteria E7 and E9) was computed, using an empirically determined δ-ray cross section.

The δ-ray cross section was determined from 95 δ rays with energies greater than 2.5 MeV found on a sample of electron tracks from conversion pairs. The same attachment criteria (E8) and energy measurement

TABLE III. Detection efficiencies for electrons from $\Sigma^{\pm}$ decay for $P_\Sigma=250$ MeV/c.

Detection method	Sigma charge	Detection efficiency (%)	
Curl up	−	37±1.5	
Curvature	−	8±1.0	
δ ray	−	10±1.7	
Total Σ^-			55±3
Curl up	+	12±1.0	
Curvature	+	3±0.4	
δ ray	+	6±1.0	
Total Σ^+			21±2

methods were used as were used with the Σ_β-decay candidates. The result was averaged over variable film conditions which are known to affect the empirical cross section.[18] The average mean free path was (55 ± 9) cm for negative electrons and (60 ± 10) cm for positrons. The dependence of the mean free path on the minimum energy of the δ ray was assumed to be linear.[19]

The initial energies of the electrons in the sigma rest frame were chosen randomly in such a way that the usual β-decay spectrum was produced in a large number of such choices. The initial energy was then transformed into the laboratory by using the measured angle between the tracks of the sigma and its decay product and an assumed sigma momentum. Varying this assumed momentum between 100 and 400 MeV/c was found to vary the detection efficiency by only 1%. A drastic change in the shape of the β-decay energy spectrum had a similarly negligible effect.

In order to test the basic validity of some of the methods used, the program was run once using the electron energy distribution, initial value choices, and dip criterion stated in the report of the Λ_β-decay experiment,[6] but with the random set of Σ origins from this experiment. The detection efficiency for electrons which curled up was 65%, as compared with 60% in the Λ_β-decay experiment. The program used in the latter experiment was, in turn, partially checked with $\mu^-\rightarrow e^-$ decays.

The efficiencies in the various categories are tabulated in Table III. The errors in these efficiencies are the result of the statistical error arising from the fact that only 180 sigma origins were used and the result of the uncertainty in the determination of the δ-ray cross section. The efficiencies are much smaller for $\Sigma_\beta{}^+$ decay because of the requirement that the positron have a measured momentum greater than 53 MeV/c (see E4). The detection efficiency as a function of energy, along with the theoretical energy spectrum for Σ_β decay and the energies of the 11 events in this experiment are shown in Fig. 2.

If the detection efficiencies for each category are multiplied by the appropriate scanning efficiencies

[18] The cross section increases with the bubble density of a minimum ionizing track. See Ref. 6.

[19] B. Rossi, Ref. 10, p. 16.

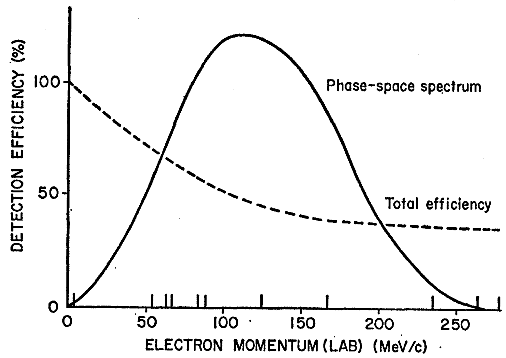

FIG. 2. Theoretical electron momentum spectrum for Σ_β decay and detection efficiency of the chamber for electrons as a function of energy from the Monte Carlo program. The spectrum curve was obtained by transforming the usual β-decay phase-space spectrum from the Σ rest frame into the laboratory at three Σ momenta (100, 250, and 400 MeV/c) and averaging the results. The solid bars indicate the momenta of the events in this experiment.

(above), the combined scanning and detection efficiencies are $(42\pm3)\%$ for Σ_β^- decays and $(16\pm2)\%$ for Σ_β^+ decays.

The Branching Ratio

The total number of events, corrected for background, is divided by the average detection and scanning efficiency and the number of mesonic sigma decays to obtain the Σ_β^--decay branching ratio:

$$\frac{\Gamma(\Sigma^-\rightarrow n+e^-+\bar{\nu})}{\Gamma(\Sigma^-\rightarrow n+\pi^-)}=\frac{9}{(0.42)\times(22\,000)}=(1.0_{-0.3}^{+0.4})\times10^{-3}. \quad (6)$$

Only 15% of the error comes from factors other than the statistics of 11 events.

For the Σ_β^+-decay branching ratio, only an upper limit can be set:

$$\frac{\Gamma(\Sigma^+\rightarrow n+e^++\nu)}{\Gamma(\Sigma^+\rightarrow n+\pi^+)}<\frac{1}{(0.16)\times(17\,000)}=0.4\times10^{-3}. \quad (7)$$

The upper limit on the ratio of the two branching ratios is

$$\frac{\Gamma(\Sigma^+\rightarrow n+e^++\nu)}{\Gamma(\Sigma^-\rightarrow n+e^-+\nu)}<0.4. \quad (8)$$

The 90% confidence limit for the same ratio is 0.6.

DISCUSSION

Several points on the internal consistency of the data are worth noting. The ratio of the detection efficiencies for the three methods of detection (3.7:0.8:1) is quite consistent with the ratio of the number of events observed in each category (3.5:1:1). From Figs. 1 and 2 it can be seen that the length and energy distributions of the 11 events are consistent with what is expected.

The upper limit of the ratio of the Σ^+ and Σ^- decay rates [Eq. (8)] may be obtained in another way by counting only those Σ_β^- decays in which the electron had a momentum significantly greater than 53 MeV/c, as defined in (E4). There were 6 such events, or 5 after subtracting one as probably background. Thus, the result of the detection efficiency program is replaced by information from the experimental electron energy spectrum. The ratio is simply

$$\frac{\Gamma(\Sigma^+\rightarrow n+e^++\nu)}{\Gamma(\Sigma^-\rightarrow n+e^-+\bar{\nu})}<\frac{1}{5}\times\frac{22\,800}{17\,200}=0.3, \quad (9)$$

compared with 0.4 from Eq. (8).

The existence of a scanning bias against high-energy δ rays, a possibility which has already been mentioned, would increase the Σ_β^--decay rate only 10%. No other systematic biases considered would have more than a 5% effect. An examination of the rescanning revealed no scanning bias dependent on the charge of the event.

CONCLUSIONS

The ratio of the rate for $\Delta S=-\Delta Q$ transitions to the rate for $\Delta S=+\Delta Q$ transitions in Σ_β decays found in this experiment is less than 0.4. Neither this experiment nor any other experiment to date has produced an example of the decay $\Sigma^+\rightarrow n+e^++\nu$. The background calculations made in this experiment indicate that it would be fruitful to continue to search for this decay mode in heavy liquid chambers even if the rate is ten times smaller than the limit quoted here.

The Σ_β^--decay branching ratio was found to be $(1.0_{-0.3}^{+0.4})\times10^{-3}$. This is 60 times smaller than the rate predicted by Feynman and Gell-Mann.[1] The Λ_β-decay rate was found to be smaller than the prediction by about the same factor.[6]

The rate determined in this experiment is not inconsistent with the tentative result of the Cern-Maryland experiment[5] in liquid hydrogen.

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Some Inequalities for the Forward Scattering Amplitude*

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We develop a class of inequalities involving the real parts of the forward scattering amplitude at arbitrary energies and certain finite integrals over total cross sections. These should prove useful in the resolution of phase shift ambiguities in the analyses of scattering data and are sufficiently flexible to be applied even in situations where, for instance, the magnitude of the residue at a pole is not known.

I. INTRODUCTION

SINCE the first development of the subject by Goldberger and others,[1] forward dispersion relations have been extensively used in the phenomenological analyses of π-N,[2] K-N,[3] and N-N[4] scattering data. The technique has had a somewhat limited success, however, due to the presence of three adverse factors: (a) ignorance of total cross sections in the high-energy region, (b) ignorance of the magnitudes of the residues at certain poles, and (c) the presence of unphysical regions in the dispersion integrals. The dispersion relations have hitherto been used as identities for the real parts of the forward scattering amplitudes in terms of certain integrals over total cross sections. We wish to demonstrate in this paper that if these are converted into suitable inequalities, the factors (a) and (b) need no longer be problems while the difficulties associated with the factor (c) can be rendered much less severe. It is, however, true that in this process, we lose some of the information which is in principle contained in the canonical identities.

Section II begins with a derivation of these inequalities for $\pi^{\pm}-p$ scattering under very weak assumptions regarding the asymptotic behavior of the amplitude. These results are then extended by observing that the $\pi^{-}-p$ total cross section seems to be always somewhat larger than the $\pi^{+}-p$ total cross section beyond a certain energy. Still further inequalities can be proved if we are willing to assume that the real part of the amplitude does not increase like energy itself at large energies. Evidence for this assumption is somewhat ambiguous, but it seems intuitively rather plausible.

Section III is concerned with the $K^{\pm}-p$ system which is a typical example where both unknown coupling constants and unphysical regions occur. We show how inequalities can be written down which do not involve coupling constants provided their signs are known. These signs depend essentially only on the relative parities of the particles involved and are rather well established for this system.[5] The section concludes with some remarks on the unphysical region.

The Appendix describes how one can derive a whole class of simpler inequalities from the previous results and follows the discussion of a similar problem by the author in a different context.[6]

The extension of the foregoing considerations to other scattering processes is fairly trivial and is not therefore considered in this paper.

For the benefit of the reader interested primarily in the results, we mention that these are contained in Eqs. (II.9), (II.16), (II.23), (III.4), (III.6)–(III.10), and the Appendix. The principal conventions about

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[1] M. Gell-Mann, M. L. Goldberger, and W. Thirring, Phys. Rev. **95**, 1612 (1954); M. L. Goldberger, *ibid.* **99**, 979 (1955); M. L. Goldberger, H. Miyazawa, and R. Oehme, *ibid.* **99**, 986 (1955).

[2] H. L. Anderson, W. C. Davidon, and U. E. Kruse, Phys. Rev. **100**, 339 (1955); U. Haber-Schaim, *ibid.* **104**, 1113 (1956); T. Spearman, Nucl. Phys. **16**, 402 (1960); F. Salzman and G. Salzman, Phys. Rev. **120**, 599 (1960). See G. Puppi, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1958), p. 43 for further references.

[3] See R. H. Dalitz, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1958), p. 191 and R. Karplus, L. Kerth and T. Kycia, Phys. Rev. Letters **2**, 510 (1959).

[4] Riazuddin, Phys. Rev. **121**, 1509 (1961).

[5] A summary of the evidence and detailed references are to be found in G. A. Snow, *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 795.

[6] A. P. Balachandran, EFINS-63-67 (unpublished); J. Math. Phys. (to be published).